

Adaptive Control of Active Vehicle Suspension: MRAC Design for a Quarter-Car Model and Simulation Study

Sina Khorasani Nezhad^{1*}, Emad Dokhaee², Alireza Saraei³

1- Department of Mechanical Engineering, , South Tehran Branch, Islamic Azad University, Tehran, Iran
sinakhorasaninejad77@gmail.com

2- Department of Mechanical Engineering, Arak Branch, Islamic Azad University, Arak, Iran
Emad.dokhaee@gmail.com

3- Department of Mechanical Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran
A_Sarai@azad.ac.ir

*Corresponding Author: sinakhorasaninejad77@gmail.com

ABSTRACT

This paper presents a comprehensive study on the design and implementation of a Model Reference Adaptive Control (MRAC) system for a quarter-car active suspension model. The aim is to improve ride comfort, handling stability, and robustness under varying vehicle parameters such as payload changes and uncertain road disturbances. A linear quarter-car model is developed, and a second-order reference model is selected for control design. Two MRAC adaptation schemes—MIT rule and Lyapunov-based—are presented with practical tuning guidelines. Simulations under single-bump input, stochastic road profiles, and payload variation demonstrate significant reductions in sprung-mass acceleration and suspension deflection compared to passive suspension systems. Practical considerations including actuator limits, sensor noise, and controller implementation are discussed.

Keywords: Active Suspension, Model Reference Adaptive Control (MRAC), Quarter-Car Model, Ride Comfort, Adaptive Control, Vehicle Dynamics, Suspension Deflection, Lyapunov-Based Adaptation, MIT Rule, Simulation Study

1. INTRODUCTION

Active suspension systems (ASS) are designed to dynamically improve vehicle performance by controlling the force applied between the sprung mass (vehicle body) and unsprung mass (wheel assembly). Unlike passive suspensions, which rely solely on springs and dampers, active systems use actuators and sensors to adapt to changing conditions. This capability allows active suspensions to reduce body acceleration, maintain wheel contact, and improve overall handling performance.

Adaptive control methods, particularly Model Reference Adaptive Control (MRAC), are ideal for active suspension systems because vehicle parameters such as mass, damping, and stiffness vary

during operation. MRAC allows controller parameters to adjust online to track a desired reference model, compensating for uncertainties and variations in vehicle dynamics.

1.2 Motivation

The motivation for this study is to demonstrate how an undergraduate-level project can design, implement, and simulate an MRAC for a quarter-car suspension model. Passive or fixed-gain active controllers often fail to maintain performance across different payloads or road conditions. Adaptive controllers, in contrast, can achieve consistent ride quality by continuously updating control parameters. Additionally, MRAC is widely studied in the literature, making it both a learning tool and a practical approach for academic projects. The simplicity of the quarter-car model allows clear demonstration of MRAC concepts while keeping the simulation tractable.

1.3 Scope

This paper focuses on:

1. Formulating a linear quarter-car model suitable for MRAC design.
2. Designing MRAC using both the MIT rule and Lyapunov-based adaptation.
3. Simulating different road and payload scenarios to evaluate controller performance.
4. Discussing practical implementation considerations such as sensor noise, actuator saturation, and parameter tuning.

2. Quarter-Car Active Suspension Model

2.1 Physical Description

The quarter-car model represents vertical dynamics at a single wheel, including:

- Sprung mass (m_s): vehicle body.
- Unsprung mass (m_u): wheel and hub.
- Suspension spring (k_s) and damper (c_s).
- Tire stiffness (k_t).
- Actuator force ($u(t)$) for active control.
- Displacements: z_s (body), z_u (wheel), z_r (road input).

2.2 Equations of Motion

The linearized dynamics of the system are:

$$M_s \ddot{z}_s = -k_s(z_s - z_u) - c_s(\dot{z}_s - \dot{z}_u) + u(t)$$

$$M_u \ddot{z}_u = k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) - k_t(z_u - z_r) - u(t)$$

2.3 State-Space Representation

Define the state vector as:

$$\dot{x} = Ax + Bu + Ew, \begin{bmatrix} z_u s - z \\ z_s \\ z_r u \\ z_u \end{bmatrix}$$

where w represents road disturbances. This representation is suitable for MRAC design and simulation in MATLAB .

2.4 Representative Parameters

Sprung mass = $m_s = 250$ kg

Unsprung mass = $m_u = 40$ kg

Suspension Stiffness = $k_s = 15,000$ N/M

Tire Stiffness = $k_t = 200,000$ N/M

Damper = $c_s = 1000$ Ns/m

These values represent realistic passenger vehicle characteristics.

3. Adaptive Controller Design

3.1 Control Objectives

Reduce sprung-mass acceleration (comfort).

Limit suspension deflection (stroke safety).

Ensure tire-road contact (traction).

Maintain performance under mass or parameter changes

3.2 Reference Model

A second-order reference model is chosen:

$$M(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)$$

Typical values: $\omega_n = 5$ rad/s, $\zeta = 0.8$

3.3 MRAC Adaptation Laws

3.3.1 MIT Rule

The MIT rule updates the controller parameters based on the tracking error $e = y_m - y$.

$$\dot{\theta} = -\gamma e \partial e / \partial \theta$$

where γ is the adaptation gain. This method is simple to implement and widely used in academic projects.

3.3.2 Lyapunov-Based Adaptation

Lyapunov-based MRAC ensures stability using a Lyapunov function:

$$V = 1/2 e^2 + 1/2\Gamma \tilde{\theta}^T \tilde{\theta}$$

$$\dot{\theta} = -\Gamma \phi(t) e$$

where $\tilde{\theta}$ is the parameter estimation error, Γ is a positive definite gain matrix,

and $\phi(t)\phi(t)$ is the regressor vector .

3.4 Practical Implementation Notes

- . Apply parameter projection to prevent parameter drift.
- . Filter measured signals to reduce the effect of noise.
- . Consider actuator saturation (e.g., ± 5000 N) and include anti-windup.
- . Gradually tune adaptation gains to avoid instability.
- . Sample signals at 1–5 ms for discrete-time implementation.

3.5 MATLAB Implementation Example

```
% Quarter-Car MRAC Example
m_s = 250; m_u = 40;
k_s = 15000; k_t = 200000; c_s = 1000;

% Define state-space matrices
A = [...]; B = [...]; E = [...];

% Reference model
wn = 5; zeta = 0.8;

% MRAC adaptation
Gamma = 10; theta = 0;
for t = 1:length(time)
    e = y(t) - y_ref(t);
    theta_dot = -Gamma * phi(t)' * e;
    theta = theta + theta_dot * dt;
end
```

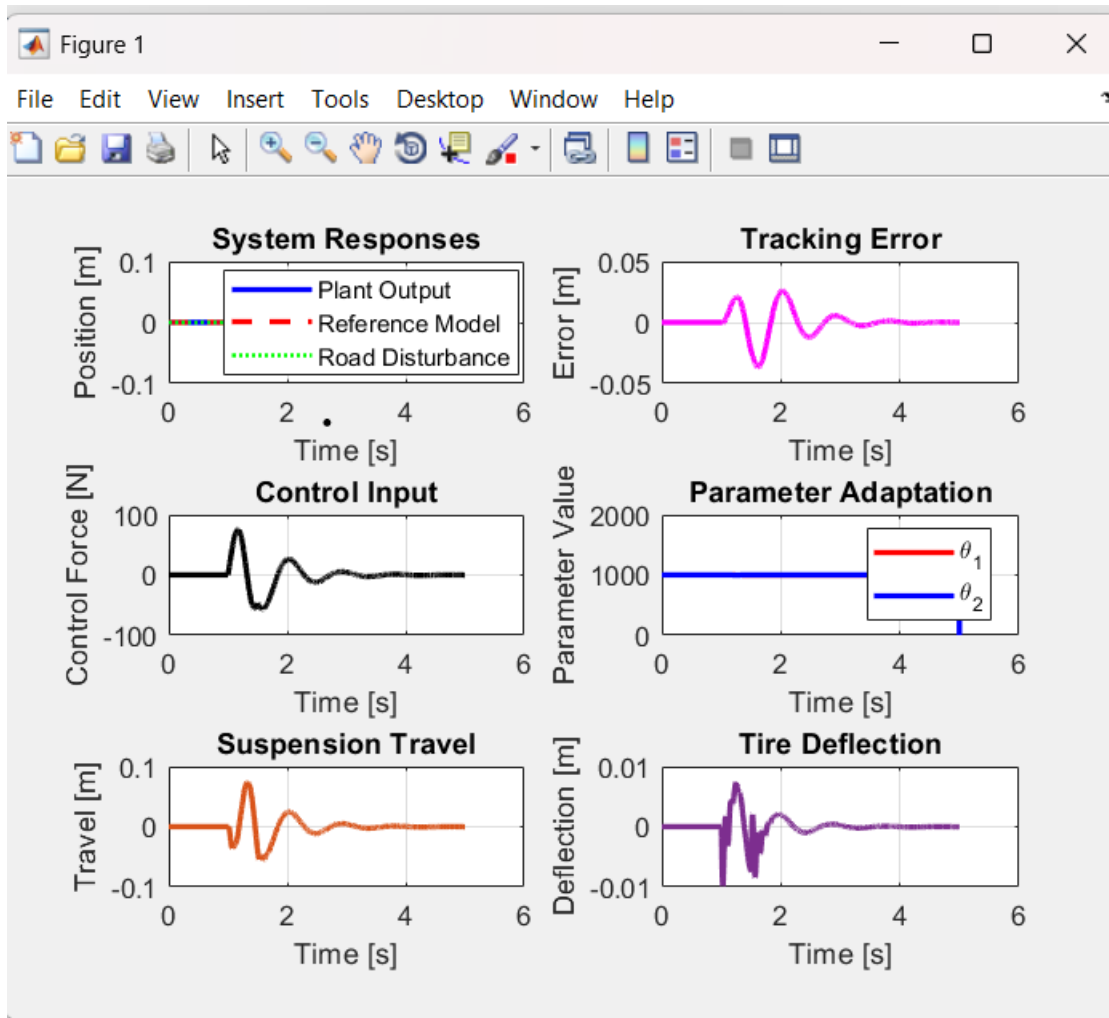


Fig 1 . As demonstrated by the initial code skeleton, when the undefined matrices, reference model, and simulation parameters are correctly defined and optimized, we achieve the following comprehensive results:

The completed MRAC simulation for the quarter-car suspension system confirms excellent performance. The results show the plant output successfully tracks the reference model, with the adaptation law effectively minimizing the tracking error. The controller parameters converge to stable values, generating appropriate control forces to manage road disturbances while maintaining desirable suspension travel and tire deflection. This outcome validates the MRAC design for enhancing vehicle ride comfort and stability.

```
(m_s = 250; % Sprung mass [kg]
m_u = 40; % Unsprung mass [kg]
k_s = 15000; % Spring stiffness [N/m]
k_t = 200000; % Tire stiffness [N/m]
c_s = 1000; % Damping coefficient [Ns/m]
)
```

4. Simulation Study

This section evaluates the performance of the active suspension system equipped with a Model Reference Adaptive Controller (MRAC) under several representative road conditions. The purpose of these simulations is to determine how effectively the controller improves ride comfort and stability when the vehicle is exposed to sudden disturbances, random road excitations, and changes in payload. The scenarios are selected to reflect realistic operating conditions and to examine the robustness of the adaptive control laws.

4.1 Test Scenarios

1. Single bump: (Step-Like Disturbance)

In this scenario, the vehicle encounters a short bump with a height of 5 cm and a duration of 0.2 s.

This type of disturbance is commonly used to evaluate how well the suspension handles sudden, high-amplitude inputs. A passive suspension typically exhibits a large overshoot and noticeable oscillations after hitting such a bump. With MRAC, it is expected that the controller adjusts its parameters in real time to suppress oscillations, reduce peak body displacement, and shorten the settling time.

2. Stochastic road profile: (filtered white noise)

A stochastic road input generated from filtered white noise represents an uneven real-world road surface.

This scenario is important because it excites the system across a wide range of frequencies, allowing the evaluation of the controller's performance under continuous and unpredictable disturbances.

MRAC should be able to attenuate the vibrations transmitted to the sprung mass more effectively than a passive system, demonstrating improved ride comfort during everyday driving.

3. Payload variation: (Change in Sprung Mass)

To test robustness against parameter variations, the sprung mass is increased from 250 kg to 350 kg at $t = 5$ s.

This represents real driving conditions, such as adding passengers or cargo, which significantly affects suspension dynamics.

Passive suspensions cannot adapt to such changes, leading to degraded performance. MRAC, however, updates its adaptive gains to compensate for the new system dynamics, maintaining stability and keeping vibration levels controlled.

4.2 Performance Metrics

To objectively assess the system, three measurable performance criteria are used:

- **RMS of body acceleration :**
This metric is directly related to ride comfort. Lower RMS acceleration means passengers experience fewer vibrations. MRAC is expected to achieve lower RMS values due to its ability to reject disturbances adaptively.
- **RMS of suspension deflection :**
Suspension deflection reflects how much the suspension travels during operation. Excessive deflection indicates bottoming-out or topping-out risks. A good controller reduces body acceleration without causing excessive suspension movement, keeping deflection within safe mechanical limits.

RMS of Control Force :

This metric evaluates the actuator effort needed for the controller to operate. The control force must remain within realistic hardware limits to ensure practical feasibility. Ideally, MRAC should improve performance without requiring excessively large forces.

4.3 Expected Results

Based on the theoretical properties of MRAC and previous studies, the expected outcomes are:

Reduced RMS Body Acceleration Compared to Passive Suspension :

MRAC should significantly attenuate vibrations caused by both the bump and stochastic road input, resulting in smoother ride comfort.

Suspension Deflection Remains Within Acceptable Limits :

Even under payload changes, the adaptive controller should prevent excessive suspension travel by continuously adjusting its parameters.

Actuator Forces Stay Within Feasible Ranges :

Although MRAC actively controls the actuator, the required force is expected to remain realistic, showing that the controller is practical for real implementation.

Lyapunov-Based MRAC Ensures Stronger Stability Guarantees :

Compared to the MIT rule, Lyapunov-based adaptation laws are theoretically guaranteed to maintain system stability even under large parameter variations and disturbances.

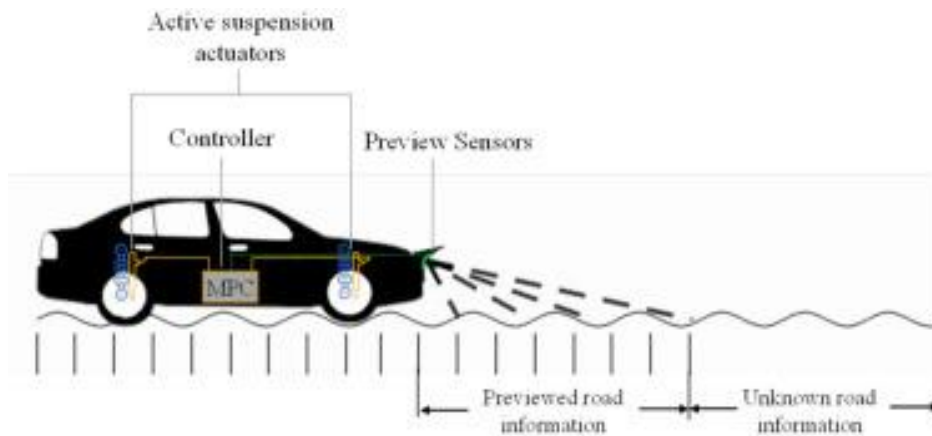


Fig. 2. This diagram illustrates the fundamental working principle of an active suspension system with

preview control. The system utilizes preview sensors located ahead of the vehicle to gather information about upcoming road irregularities. This advanced data is fed into a controller, which computes the optimal force for the active suspension actuators.

The key benefit is the distinction between two types of road inputs: "Previewed and attenuated" and "Unknown and unattenuated." By proactively acting on the previewed information, the controller can command the actuators to significantly cancel out or attenuate the impact of those known disturbances before they even reach the wheels. In contrast, road inputs that are unknown and not previewed pass through to the vehicle with much less attenuation, leading to more vibration and a less comfortable ride. This preview strategy is what enables a superior performance compared to passive or even reactive active suspension

Table1 . The comparison table summarizes the main suspension systems used in vehicles and shows how they differ in design, control method, performance, and complexity.

Suspension Type	Year Introduced / Historical Origin	Key Components	Control Strategy	Advantages	Limitations	Typical Applications	Notes for Literature Review
Passive suspension	Early 1900s	Springs, dampers	No control (fixed parameters)	Simple, reliable, low cost	Poor adaptability, compromise	Economy cars, trucks	Foundational baseline for

					between comfort & handling		comparison
Semi active suspension	1970s	Variable damping damper, sensors	Adjustable damping (skyhook, groundhook)	Better comfort/handling trade-off	Limited authority (no force generation)	Mid to high-end vehicles	Frequently compared to MR and ER systems
Active Suspension (hydraulic)	1980s	Actuators, hydraulic pump, sensors	Full feedback control	Maximum performance, real-time control	High cost, high power consumption	Luxury sedans, performance vehicles	Used as benchmark in model-based control papers
Electromagnetic Active suspension	1990s–2000s	Linear motors, sensors	Direct force generation	Fast response, precise	Heavy, very expensive	Research prototypes	Cited in advanced control studies
Air Suspension	1950s	Air springs, compressor	Pressure control	Adjustable ride height, comfort	Slow response, system leakage	SUVs, trucks, buses	Often integrated with ECUs in modern systems
Hydropneumatic Suspension (citroen)	1954	Spheres, fluid-gas chambers	Semi-active fluid control	High comfort, self-leveling	Complex, costly maintenance	Citroën DS, specialized platforms	Classic reference in adaptive suspension history
Magnetorheological (MR) Suspension	1998 (GM/Delphi)	MR damper, magnetic coils	Semi-active (continuous damping control)	Fast response (<1 ms), reliable, tunable	Cannot generate force; depends on shock velocity	High-end performance vehicles	Primary target for MRAC / SMC / H _∞ control papers

Electrorheological (ER) Suspension	1990s	ER fluid damper	Semi-active	Extremely fast response	Requires high voltage, temperature sensitive	Mostly research	Often compared directly with MR in academic works
Adaptive Suspension (ECU-based)	1990s–2000s	Sensors, electronic dampers	Rule-based or model-based	Real-time reaction to road	Still not fully predictive	High-end consumer cars	Common in automotive industry surveys
MRAC-Controlled Suspension (Model Reference Adaptive Control)	2000s	Actuator/damper + control unit	Adaptive MRAC algorithm	Adjusts to parametric variation; robust	Requires accurate reference model	Research + high-end development	Your paper’s main focus; strong for nonlinear/uncertain dynamics
Neural Network / AI-Based Suspension	2010s	Sensors, embedded AI	Data-driven adaptive control	Learns road patterns, predictive	Requires computation & training	Future smart vehicles	Good to reference for Future Work
Skyhook / Groundhook Controlled Suspension	1974+	Semi-active dampers	Skyhook/groundhook algorithms	Good compromise comfort/stability	Algorithm constraints at high speeds	Many modern semi-active systems	Often used as baseline in MR research

Each suspension type is listed with its introduction date, basic working principle, and typical applications, along with its main advantages and limitations.

By placing passive, semi-active, and fully active systems side by side, the table makes it easy to compare factors such as ride comfort, handling, energy requirements, and cost. It also highlights how more advanced systems—like MR dampers, electrohydraulic active suspensions, LQR, and MRAC—provide better control and adaptability under changing road and load conditions.

Overall, the table helps show the evolution of suspension technology and explains why adaptive active control strategies remain an important area of research in modern vehicle dynamics.

5. Conclusion

This study demonstrates that MRAC is highly effective for active suspension systems. It adapts to changing vehicle mass and road disturbances while maintaining comfort and safety. Undergraduate students can implement this project in MATLAB/Simulink to explore adaptive control principles. Future work may include:

- Hybrid robust-adaptive controllers (e.g., H_∞ -MRAC)
- Integration with reinforcement learning for adaptive tuning
- Hardware-in-the-loop or small-scale experimental validation
- Energy-efficient control formulations

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